## Problem Set 9 (Due: Nov. 23, 2011)

November 16, 2011

Your homework should be submitted to TA (Chih-Han Lin) at the beginning of the course. Generally, You have a week to finish the problem set since it was revealed. If you cannot submit it on time, you can still bring your homework to TA's office (L325, Institute of Atomic and Molecular Sciences, in NTU campus) and put it into TA's mailbox which is on the top of the shoe cabinet in L325. Always remember to remind TA to check your homework by e-mail (clin@ltl.iams.sinica.edu.tw) if you don't submit it in class.

## Ex 1

For a dynamical observable $\mathbf{A}(t)$ which is time dependent explicitly, the time derivative of the expectation value takes the form as

$$
i \hbar \frac{d}{d t}\langle\mathbf{A}(t)\rangle_{t}=\langle[\mathbf{A}, \mathbf{H}]\rangle_{t}+i \hbar\left\langle\frac{\partial \mathbf{A}}{\partial t}\right\rangle_{t} .
$$

Use the above formula repeatedly and show that

$$
\langle\Delta \boldsymbol{x}\rangle_{t}^{2}=\langle\Delta \boldsymbol{x}\rangle_{0}^{2}+\frac{2}{m}\left\{\frac{1}{2}\langle\boldsymbol{x} \boldsymbol{p}+\boldsymbol{p} \boldsymbol{x}\rangle_{0}-\langle\boldsymbol{x}\rangle_{0}\langle\boldsymbol{p}\rangle_{0}\right\} t+\frac{\langle\boldsymbol{p}\rangle_{0}^{2}}{m^{2}} t^{2},
$$

for the case of a free particle in one-dimension.

## Ex 2

Prove that the one dimensional Schrödinger equation in free space

$$
i \hbar \frac{\partial}{\partial t} \psi(x, t)=-\frac{\hbar^{2}}{2 m} \frac{\partial^{2}}{\partial x^{2}} \psi(x, t)
$$

is invariant under Galilean transformation:

$$
\begin{aligned}
x^{\prime} & =x-v_{0} t, \\
t^{\prime} & =t .
\end{aligned}
$$

(Hint: let $\left.\psi^{\prime}\left(x^{\prime}, t^{\prime}\right)=f\left(x, t ; v_{0}\right) \psi(x, t).\right)$

## Ex 3

Consider the fomula for $\mathbf{U}\left(t, t_{0}\right)$ expressed as

$$
\begin{align*}
\mathbf{U}\left(t, t_{0}\right) & =\mathcal{T}\left\{\exp \left(-\frac{i}{\hbar}\right) \int_{t_{0}}^{t} d t \mathbf{H}(\mathbf{P}, \mathbf{Q} ; t)\right\} \\
& =\mathbf{I}+\left(-\frac{i}{\hbar}\right) \int t^{\prime} \mathbf{H}\left(t^{\prime}\right)+\frac{1}{2!}\left(-\frac{i}{\hbar}\right)^{2} \int d t^{\prime} d t^{\prime \prime} \mathcal{T}\left(\mathbf{H}\left(t^{\prime}\right), \mathbf{H}\left(t^{\prime \prime}\right)\right) \\
& +\frac{1}{3!}\left(-\frac{i}{\hbar}\right)^{3} \int d t^{\prime} d t^{\prime \prime} d t^{\prime \prime \prime} \mathcal{T}\left(\mathbf{H}\left(t^{\prime}\right), \mathbf{H}\left(t^{\prime \prime}\right), \mathbf{H}\left(t^{\prime \prime \prime}\right)\right)+\ldots \tag{1}
\end{align*}
$$

if $\mathbf{H}(\mathbf{P}, \mathbf{Q} ; t) \equiv \mathbf{H}(\mathbf{P}, \mathbf{Q})$, namely $\mathbf{H}$ does not depend on time explicitly, show that $\mathbf{U}\left(t, t_{0}\right)$ reduces to the usual expression as

$$
\begin{equation*}
\mathbf{U}\left(t, t_{0}\right)=\exp \left(-\frac{i}{\hbar} \mathbf{H}(\mathbf{P}, \mathbf{Q})\left(t-t_{0}\right)\right) \tag{2}
\end{equation*}
$$

