

Problem Set 4 (Due: Oct. 19, 2011)

October 12, 2011

Your homework should be submitted to TA (Chih-Han Lin) at the beginning of the course. Generally, You have a week to finish the problem set since it was revealed. If you cannot submit it on time, you can still bring your homework to TA's office (L325, Institute of Atomic and Molecular Sciences, in NTU campus) and put it into TA's mailbox which is on the top of the shoe cabinet in L325. Always remember to remind TA to check your homework by e-mail (clin@ltml.iams.sinica.edu.tw) if you don't submit it in class.

Ex 1

Show that delta function $\delta(x)$ can be expressed as

(a)
$$\delta(x) = \frac{1}{\pi} \lim_{N \rightarrow \infty} \frac{\sin Nx}{x}.$$

(b)
$$\delta(x) = \frac{1}{2} \frac{d^2}{dx^2} |x|.$$

Ex 2

Show that

$$\langle x | \mathbf{P}^2 | x' \rangle = \left(\frac{\hbar}{i} \right)^2 \delta''(x - x'),$$

and hence generalize to

$$\langle x | F(\mathbf{P}) | x' \rangle = F \left(\frac{\hbar}{i} \frac{d}{dx} \right) \delta(x - x').$$

Ex 3

Consider a quantum system with a state $|\psi\rangle$ that is translated by a distance ξ by the unitary operator $\mathbf{U}(\mathbf{P}; \xi) = \exp(-i\xi\mathbf{P}/\hbar)$, i.e.

$$|\psi'\rangle = \mathbf{U}(\mathbf{P}; \xi) |\psi\rangle = e^{-\frac{i}{\hbar} \xi \mathbf{P}} |\psi\rangle.$$

Find the q-representation of $|\psi'\rangle$, which is the wave function $\psi'(x) = \langle x | \psi' \rangle = \langle x | \mathbf{U}(\mathbf{P}; \xi) | \psi \rangle$, explicitly and interpret the result.