Problem Set 2 (Due: Oct. 5,2011)

September 28, 2011

Your homework should be submitted to TA (Chih-Han Lin) at the beginning of the course. Generally, You have a week to finish the problem set since it was revealed. If you cannot submit it on time, you can still bring your homework to TA's office (L325, Institute of Atomic and Molecular Sciences, in NTU campus) and put it into TA's mailbox which is on the top of the shoe cabinet in L325. Always remember to remind TA to check your homework by e-mail (`clin@ltl.iams.sinica.edu.tw`) if you don't submit it in class.

Ex 1

Show that the $\|\mathbf{AB}\| \leqslant \|\mathbf{A}\|\|\mathbf{B}\|$ if $\mathbf{A}, \mathbf{B} \in$ bounded operator.

Ex 2

Let $\{\phi_1, \phi_2, \ldots, \phi_n\}$ be a set of an orthonormal basis. Prove that operator $\mathbf{U}$ is unitary if $\{\mathbf{U}\phi_1, \mathbf{U}\phi_2, \ldots, \mathbf{U}\phi_n\}$ is also a set of an orthonormal basis.

Ex 3

Show that

(a) $(\mathbf{A} + \mathbf{B})^\dagger = \mathbf{A}^\dagger + \mathbf{B}^\dagger$.

(b) $(\alpha\mathbf{A})^\dagger = \alpha^*\mathbf{A}^\dagger$.

(c) $(\mathbf{AB})^\dagger = \mathbf{B}^\dagger\mathbf{A}^\dagger$.

(d) $(\mathbf{A}^\dagger)^\dagger = \mathbf{A}$.

(e) $(\mathbf{A}^\dagger)^{-1} = (\mathbf{A}^{-1})^\dagger$.

Ex 4

Show that $\mathbf{P}_{\mathcal{M}}\mathbf{P}_{\mathcal{M}_\perp} = \mathbf{P}_{\mathcal{M}_\perp}\mathbf{P}_{\mathcal{M}} = \mathbf{O}$ (null vector).

Ex 5

Consider a Hilbert space spanned by a Hermitian operator $\mathbf{A}$.

(a) Prove that $\prod_a (\mathbf{A} - a\mathbf{I})$ is a null operator if $\mathbf{A}\psi_a = a\psi_a$.

(b) What is the significance of the following operator, $\prod_{a \neq a'} \frac{\mathbf{A} - a\mathbf{I}}{a' - a}$?