Problem Set 10 (Due: Nov. 30, 2011)

November 24, 2011

Your homework should be submitted to TA (Chih-Han Lin) at the beginning of the course. Generally, You have a week to finish the problem set since it was revealed. If you cannot submit it on time, you can still bring your homework to TA's office (L325, Institute of Atomic and Molecular Sciences, in NTU campus) and put it into TA's mailbox which is on the top of the shoe cabinet in L325. Always remember to remind TA to check your homework by e-mail (`clin@ltl.iams.sinica.edu.tw`) if you don't submit it in class.

Ex 1

By using Mehler's formula

$$\frac{1}{\sqrt{1-\xi^2}}\exp\left[-\frac{x^2+y^2-2xy\xi}{\sqrt{1-\xi^2}}\right] = e^{-(x^2+y^2)}\sum_n \frac{\xi^n}{2^n n!}H_n(x)H_n(y),$$

prove that the propagator in the case of the one dimensional harmonic oscillator can be expressed as

$$\begin{aligned} K(x,t;x_0,t_0) = & \sqrt{\frac{m\omega}{2\pi i\hbar\sin\omega(t-t_0)}} \\ & \times\exp\left\{\left(\frac{im\omega}{2\hbar\sin\omega(t-t_0)}\right)\left[(x^2+x_0^2)\cos\omega(t-t_0)-2xx_0\right]\right\}. \end{aligned} \tag{1}$$

(**Hint:** use

$$K(x,t;x',t') = \sum_\alpha \langle x|e^{-\frac{i}{\hbar}\mathbf{H}(t-t')}|\alpha\rangle\langle\alpha|x'\rangle = \sum_\alpha \langle x|e^{-\frac{i}{\hbar}E_\alpha(t-t')}|\alpha\rangle\langle\alpha|x'\rangle. \tag{2}$$

for $|\alpha\rangle = |n\rangle$.),

Ex 2

Derive explicitly the propagator in the p-representation, i.e.

$$\langle x|e^{-\frac{i}{\hbar}\mathbf{H}(t-t')}|x'\rangle = \int \mathcal{D}x\mathcal{D}p\exp\left(\frac{i}{\hbar}\int_{t'}^{t}\bar{L}(\tau)d\tau\right),$$

where

$$\mathcal{D}x\mathcal{D}p = \lim_{\epsilon\to 0}\prod_n \left(\frac{dx_n dp_n}{2\pi\hbar}\right), \qquad \int_{t'}^{t} \bar{L}(\tau)d\tau = \lim_{\epsilon\to 0}\epsilon\bar{L}_n,$$

with

$$\begin{aligned} \bar{L}_n &= p_n\dot{x}_n - H(x_n, p_n), \\ \dot{x}_n &= \frac{1}{\epsilon}(x_n - x_{n-1}). \end{aligned}$$