Problem Set 1 (Due: Sep. 28,2011)

September 21, 2011

Your homework should be submitted to TA (Chih-Han Lin) at the beginning of the course. Generally, You have a week to finish the problem set since it was revealed. If you cannot submit it on time, you can still bring your homework to TA's office (L325, Institute of Atomic and Molecular Sciences, in NTU campus) and put it into TA's mailbox which is on the top of the shoe cabinet in L325. Always remember to remind TA to check your homework by e-mail (`clin@ltl.iams.sinica.edu.tw`) if you don't submit it in class.

Ex 1

Consider the following set of continuous functions $f_n(x) = x^n, x \in [-1, 1]$ which spans a $\mathcal{L}^2(-1, 1)$-space. Find explicitly the first 3 orthonormal functions by the Gram-Schmidt process. What are those functions that occur to your mind?

Ex 2

Prove that the Minkowski inequality holds in Hilbert space, i.e.

$$\|\psi + \phi\| \leqslant \|\psi\| + \|\phi\|.$$

(**Hint:** take the square of either side.)

Ex 3

Prove the law of parallelogram holds in Hilbert space, i.e.

$$\|\psi + \phi\|^2 + \|\psi - \phi\|^2 = 2(\|\psi\|^2 + \|\phi\|^2).$$

Ex 4

Prove that every finite dimensional vector space is complete.
(**Hint:** since the real and complex numbers are complete.)

Ex 5

Base upon the definition of inner product in $\mathcal{V}$, show that $(\psi, 0) = (0, \psi) = 0$.

Ex 6

Show that the norm in $\mathcal{L}^2(-1,1)$-space can be defined as $\sqrt{(f(x), f(x))}$, namely $\|f(x)\| = \sqrt{(f(x), f(x))}$ or as maximum of $|f(x)|$ in $a \leqslant x \leqslant b$, i.e. $\|f(x)\| = \max\,\{|f(x)|, a \leqslant x \leqslant b\}$.