

$$\langle x | F(p) | x' \rangle = \sum_j F_j \left(\frac{\hbar}{i} \frac{d}{dx} \right)^j \delta(x-x')$$

$$= F \left(\frac{\hbar}{i} \frac{d}{dx} \right) \delta(x-x')$$

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3. Consider a quantum system with quantum state $|4\rangle$ which is translated by a distance ξ , i.e.

$$|4'\rangle = U(p; \xi)|4\rangle = e^{-\frac{i}{\hbar} \xi p} |4\rangle$$

Find the g-representation of $|4'\rangle$ explicitly and interpret your result.

$$U(p; \xi)|x\rangle = |x+\xi\rangle$$

$$\begin{aligned} \langle x | 4' \rangle &= \langle x | U | 4 \rangle = \int_{-\infty}^{\infty} \langle x | U | x' \rangle \langle x' | 4 \rangle dx \\ &= \int_{-\infty}^{\infty} \underbrace{\langle x | x + \xi \rangle}_{\delta(x-x-\xi)} \langle x' | 4 \rangle dx \\ &= \langle x - \xi | 4 \rangle \\ &= \psi(x - \xi) \end{aligned}$$

The effect of translation operator on position basis is to change $|x\rangle$ to $|x+\xi\rangle$, which is equivalent to changing the g-representation of ψ from $\psi(x)$ to $\psi(x-\xi)$. The same property holds in classical mechanics considering transformation.