Simulation of Kerr-lens modelocking Ti:Sapphire laser

Chih-Han Lin

Applied Physics Department, National Taiwan University

June 24, 2011

Refer to *Chin. Phys. B* **19** 014215 (2010) and *Opt. Express* **12** 2731(2004), pulse propagation around cavity of KLM laser can be describe by the following equation

Extended Nonlinear Scrödinger (ENLS) equation

$$\frac{\partial a(z,t)}{\partial z} = (\hat{L} + \hat{N})a(z,t),$$

$$\hat{L}a(z,t) = \frac{1}{2}(g(E_p) - 2l)a(z,t) - \frac{i}{2}\left[\beta_2 + i\frac{g(E_p)}{(\delta\omega)^2}\right]\frac{\partial^2}{\partial t^2}a(z,t),$$

$$\hat{N}a(z,t) = \underbrace{i\gamma|a(z,t)|^2 a(z,t)}_{\text{SPM}} - \underbrace{q(a(z,t))a(z,t)}_{\text{Fast SA}},$$

where $g(E_p) = \dfrac{g_0}{1 + E_p/E_{sat}}$, $q(a) = \dfrac{q_0}{1 + |a(z,t)|^2/q_{sat}}$ and $E_p \simeq \int |a(z,t)|^2 dt.$

$\hat{L}a(z,t)$ contains gain, group velocity delay (GVD) and spectrum narrowing effect. On the other hand, $\hat{N}a(z,t)$ collects nonlinear parts such as Self Phase Modulation (SPM) and fast Saturable Absorption induced by Kerr-lens effect.

ENLS equation is often solved by means of split-step Fourier method.

Corollary

If $\hat{N}$ and $\hat{L}$ commute, we have

$$a(z+\delta z, t) \simeq \exp(\hat{N}\delta z)\exp(\hat{L}\delta z)a(z,t).$$

It means that we can split the whole partial differential equation into linear part and nonlinear part. Firstly, we calculate $a_N(z+\delta z) = \exp(\hat{L}\delta z)a(z,t)$, and then get the next step $a(z+\delta z, t)$ by $a(z+\delta z, t) = \exp(\hat{N}\delta z)a_N(z+\delta z)$. Thus we can use different algorithm to solve the equation for optimization.

Since $\hat{N}$ in our model includes only constant and $f(D)$ (where $D = /\partial t$ is differential operator on t), we may efficiently calculate the $a(z+\delta z, t) = \exp(\hat{N}\delta z)$ in frequency domain, i.e. D becomes $i\omega$ for simplification.

$$\begin{aligned}
\tilde{a}(z,\omega) &= \mathcal{F}\{a(z,t)\}, \\
\frac{\partial \tilde{a}}{\partial z} = \hat{N}\tilde{a} &= \left[\frac{g(E_p)}{2} - l + \frac{i\beta_2}{2}\omega^2 - \frac{g(E_p)}{2(\delta\omega^2)}\omega^2\right]\tilde{a} = f(\omega)\tilde{a}, \\
\tilde{a}(z+\delta z,\omega) &= \exp(f(\omega)\delta z)\tilde{a}(z,\omega).
\end{aligned}$$

So that we have $a_N(z+\delta z, t) = \mathcal{F}^{-1}\{\exp[f(\omega)\delta z]\mathcal{F}\{a(z,t)\}$

Experimental Setup of P-GDD ker-lens modelocking Ti:S Laser

I will compare the simulation results with experiment data in *Chin. Phys. B* **19** 014215 (2010). The output average power is about 500 mW (1.68 nJ per pulse). The output power spectrum is shown as follows:

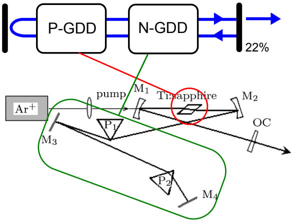


Fig. 1. Schematic of the experimental setup. M_1, M_2, dichroic spherical mirrors (R = 100 mm); M_3, M_4, flat high-reflection mirrors; P_1, P_2, fused silica prism sequence for dispersion control with an apex-to-apex distance of 950 mm; OC, 22%-transmission wedged output coupler.

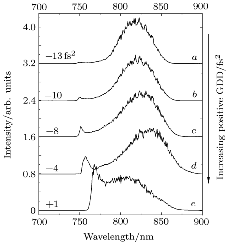


Fig. 2. Spectra with different net cavity dispersions in the vicinity of zero GDD. The GDD values are calculated at a central wavelength of 800 nm.

Negative GDD regime

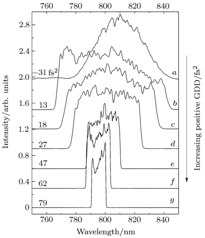


Fig. 3. Variations of power spectra with different insertions of P_2 in the positive dispersion region. The GDD values are calculated at a central wavelength of 800 nm. Curve *a*: sech^2 profiles, *b*: asymmetrical spectrum, *c*: parabolic-like spectrum, *d*: flat-top spectrum.

The simulation will focus on positive group-delay dispersion (P-GDD) regime.

The relation between GDD, spectrum width and pulse duration is shown as follows,

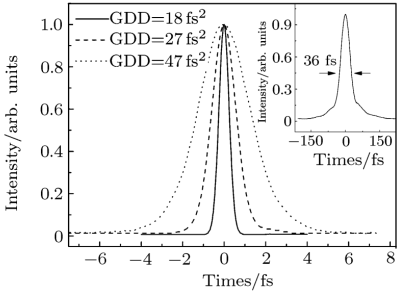


Fig. 4. Intensity autocorrelation traces of the direct output pulses with different positive GDDs. The inset shows the autocorrelation trace of the dechirped pulse (GDD=18 fs^2).

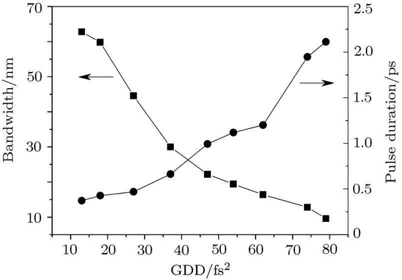


Fig. 5. Dependences of bandwidth and pulse duration on positive GDD.

RK4 method

Instead of using $\tilde{a}(z+\delta z,\omega) = \exp(f(\omega)\delta z)\tilde{a}(z,\omega)$ to calculate the next step field quantity, we can improve the precision of the numerical solution by 4 order Runge-Kutta (RK4) methods as follows:

Corollary

$$\frac{d\hat{a}}{dz} = f(\hat{a}) = f(\omega)\tilde{a},$$

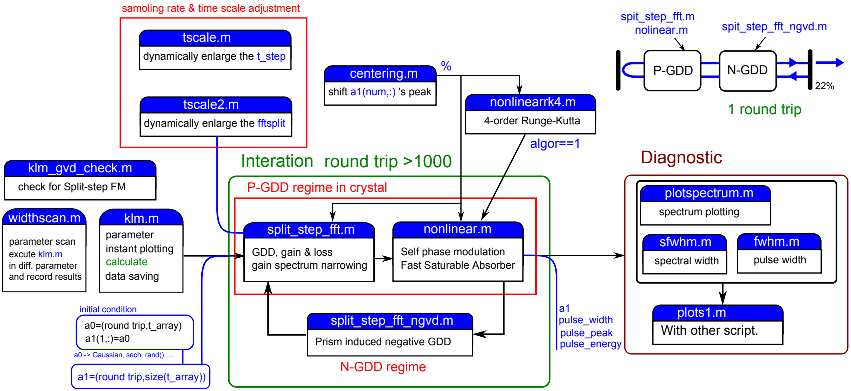
$$k_1 = \delta z f(\hat{a}(z)), \qquad k_2 = \delta z f(\hat{a}(z) + \frac{1}{2}k_1),$$

$$k_3 = \delta z f(\hat{a}(z) + \frac{1}{2}k_2), \qquad k_4 = \delta z f(\hat{a}(z) + k_3),$$

$$\hat{a}(z+\delta z,\omega) = \hat{a}(z,\omega) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

When executing nonlinear step calculation, we can promote precision by using RK4 method.

The simulation result of each round trip is stored in a 2D array `a1(cycle,size(t_index))`. The pulse width, total energy and peak intensity are also recorded and plotted after `klm.m` is finished.



quantity	value	unit	quantity	value	unit
g_0	100	m^{-1}	γ	30×10^{-5}	$(\mathrm{W}\cdot\mathrm{m})^{-1}$
E_{sat}	10	nJ	q_0	5	m^{-1}
β_2	58	$\mathrm{fs}^2/\mathrm{mm}$	q_{sat}	0.3	MW
$\delta\omega$	270	THz	$\mathrm{l}_{crystal}$	7	mm
l_{cavity}	1820	mm	η	0.22	

And the total GDD is between -50 fs^2 and 80 fs^2. $\beta_{2,prism}$ is calculated as,

$$\beta_{2,prism} = \frac{\text{total GDD} - 2\times\beta_2 l_{crystal}}{2\times(l_{cavity} - l_{crystal})}.$$

Both Simulation steps in P-GDD and N-GDD regime are tunable to get steady state solutions. `frame` and `cycle` in MATLAB code control data dump rate and total round trips.

η denotes transmission of output coupler. We can convert η into l:

$$\frac{\partial a}{\partial z} = -la \Longrightarrow a(z+\delta z) = \exp(-l\delta z)a(z) = \eta a(z),$$

$$\delta z = 2\times l_{crystal} = 14 \text{ mm} \Longrightarrow l = \frac{-\ln\sqrt{1-\eta}}{14 \text{ mm}} \simeq 0.0089 \text{ mm}^{-1}.$$

Validity of Split-Step Fourier Method

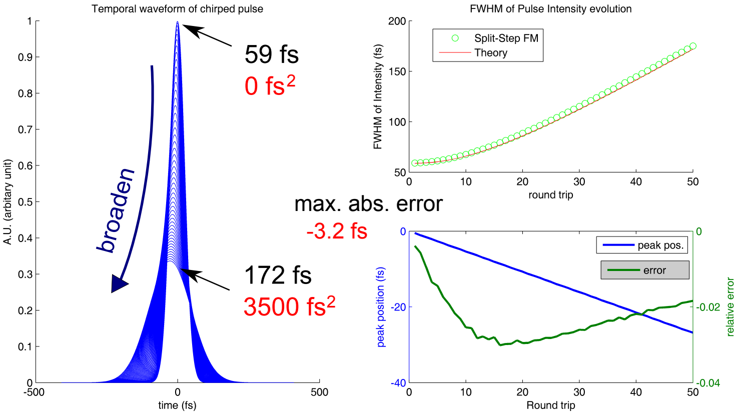
We can simulate only GVD effect and compare the pulse width broaden with analytical solution to check the validity of Split-Step Fourier method in our algorithm. Assume that the dispersion-free pulse has Gaussian shape,

$$a(t) = \exp\left(-\frac{t^2}{\tau^2}\right) \Longrightarrow I(t) = a^2(t) = \exp\left(-\frac{2t^2}{\tau^2}\right),$$
$$\frac{\partial a}{\partial z} = -\frac{iD}{2}a = \frac{i\beta_2\delta z}{2}a.$$

FWHM of I(t) is $\tau\sqrt{2\ln 2}$. If the propagation induce D=GDD, pulse width τ will be extended to τ'

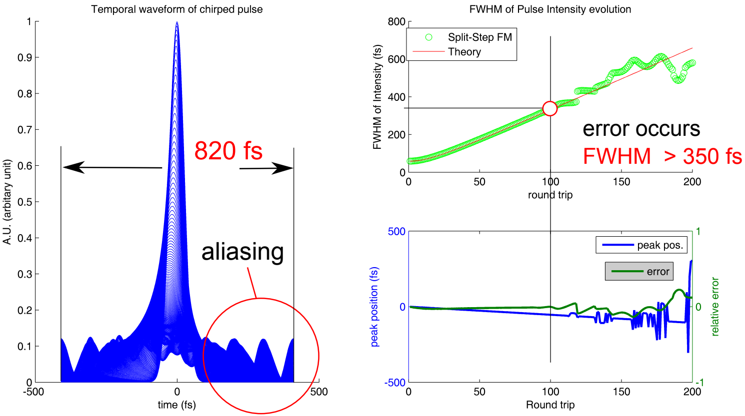
$$\tau' = \sqrt{\tau^2 + \frac{(2D)^2}{\tau^2}} = \tau\sqrt{1 + \frac{(2D)^2}{\tau^4}} \quad \text{where } D = \beta_2\delta z.$$

We use **klm_gvd_check.m** to modified the algorithm with theoretical solution. Parameters **gain_narrowing** and **gs** are temporal chosen as zero for pure GDD simulation.

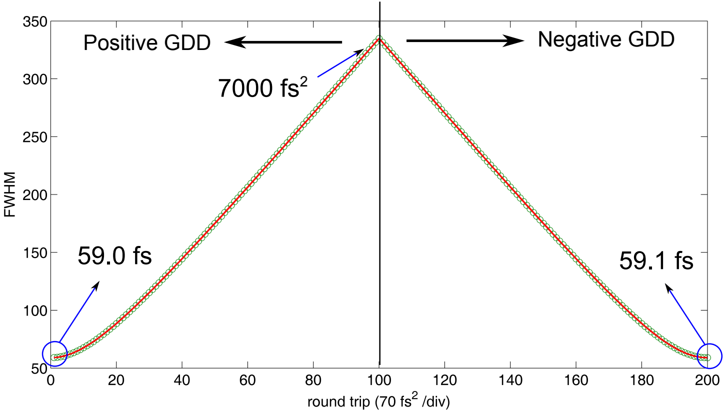


The absolute error on FWHM of Gaussian pulse is approximately -3.2 fs with $\beta_2 = 10$ fs^2/mm and round trip length $l = 7$ mm.

We need boundary conditions $\lim_{t\to\pm\infty} a(z,t) = 0$ to avoid divergence and aliasing on spectrum. In the following case, the extended pulse does not fulfill the boundary conditions and generates ripples bouncing on both end. Generally, we have to create an temporal array that is as twice larger as the FWHM of simulated pulse.

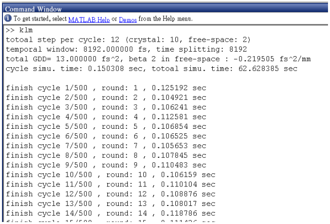


We also did another testing in GDD simulation. We first added positive GDD to broaden the pulse, and then we applied the same amount of negative GDD to compress the pulse. The calculation error $59.1 - 19.0 = 0.1$ fs dose not exceed the resolution of temporal array (0.1 fs).

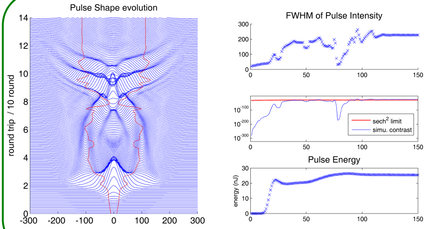


Generally the simulation will stop after 2000 round trips (10 steps in P-GDD regime and 2 steps in N-GDD regime per round trip). Here is an example for GDD=13 fs^2 simulation:

Commad widow

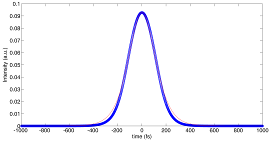


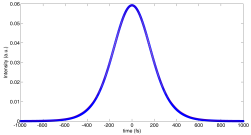
Instant plotting

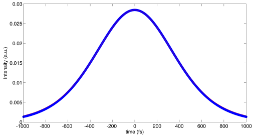


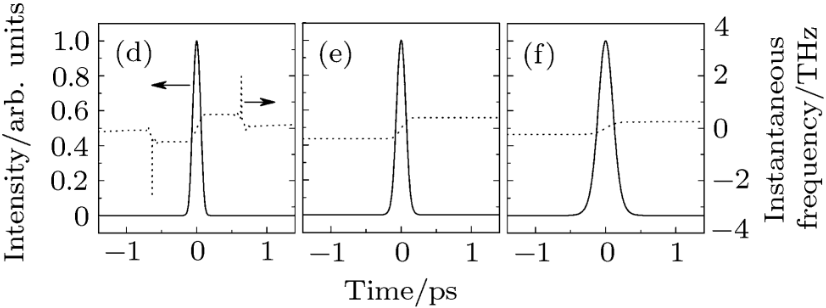
Simulated Pulse duration and Power Spectrum

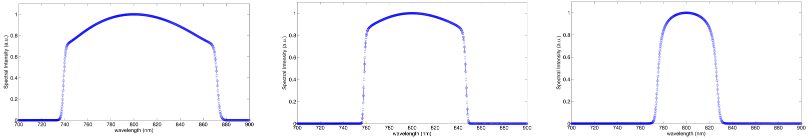
The simulation in *Chin. Phys. B* **19** 014215 (2010) chose GDD=18, 27, 47 fs^2. I did the simulation with the same parameters. Red line indicates the theoretical $sech^2$ shape function with the simulated FWHM of pulse intensity.

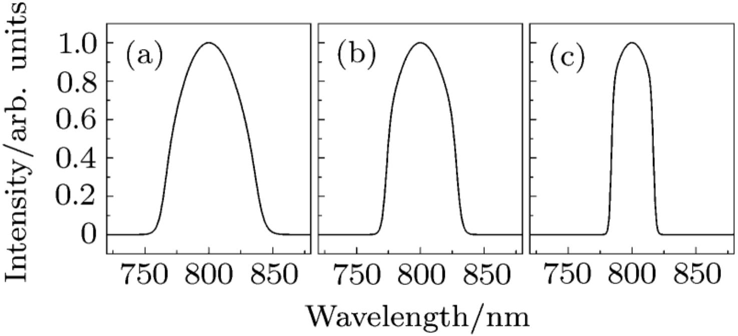




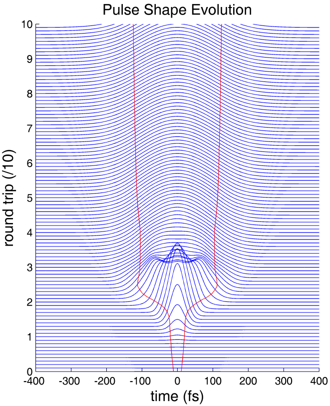


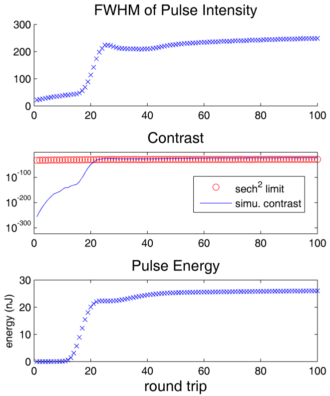



Spectral Intensity (a.u.)
wavelength (nm)
Spectral Intensity (a.u.)
wavelength (nm)
Spectral Intensity (a.u.)
wavelength (nm)

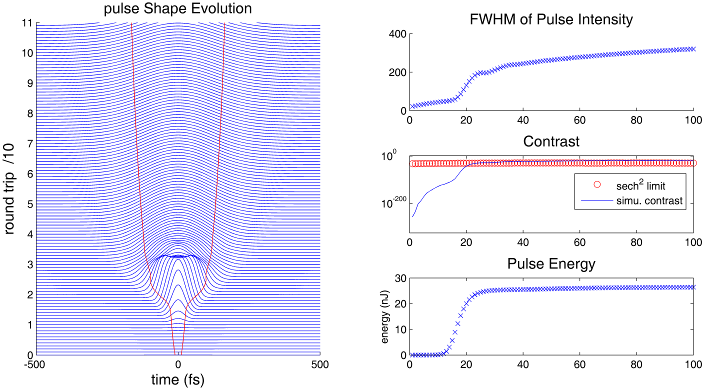

Intensity/arb. units
1.0
0.8
0.6
0.4
0.2
0
(a)
(b)
(c)
750 800 850
750 800 850
750 800 850
Wavelength/nm

total GDD=18 fs^2.

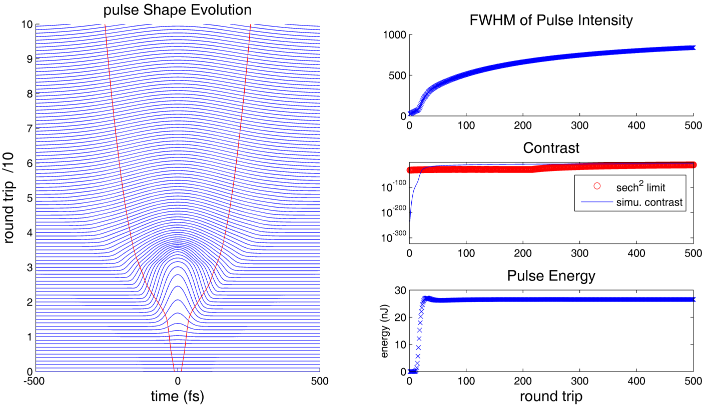


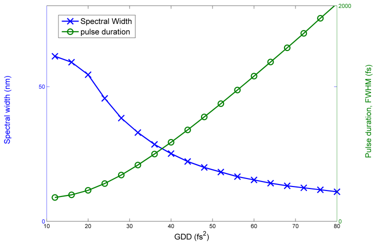


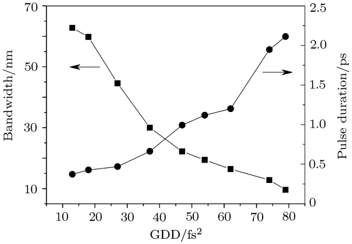
total GDD=27 fs^2.



total GDD=47 fs^2.




Spectral Width
pulse duration
Spectral width (nm)
Pulse duration, FWHM (fs)
GDD (fs^2)
2000
50
0
10
20
30
40
50
60
70
80


Bandwidth/nm
Pulse duration/ps
GDD/fs^2
70
50
30
10
2.5
2.0
1.5
1.0
0.5
0
10
20
30
40
50
60
70
80

Pulse energy is not sensitive with GDD.